C2 Jan 2007 (MA)

Q1a) $f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$

b) $\int_1^2 f(x)\,dx = \int_1^2 [x^3 + 3x^2 + 5]\,dx$

$= \left[\frac{x^4}{4} + \frac{3x^3}{3} + 5x\right]_1^2 = \left[4 + 8 + 10\right] - \left[\frac{1}{4} + 1 + 5\right]$

$= 22 - \frac{25}{4}$

$= \boxed{\frac{63}{4}}$

Q2a) $(1-2x)^5 \simeq (1)^5 + \binom{5}{1}(1)^4(-2x)^1 + \binom{5}{2}(1)^3(-2x)^2 + \binom{5}{3}(1)^2(-2x)^3$

$(1-2x)^5 \simeq 1 + 5(-2)x + 10(4)x^2 + 10(-8)x^3$

$\simeq 1 - 10x + 40x^2 - 80x^3$

b) $(1+2x)^5 \simeq 1 - 10x$

$(1+x)(1-2x)^5 \simeq (1+x)(1-10x) \simeq 1 - 10x + x \underbrace{- 10x^2}_{\text{ignore}}$

$\simeq 1 - 9x$

Q3) distance from $(-1,4)$ to $(3,6)$ = $2 \times$ radius.

distance $= \sqrt{(3--1)^2+(6-4)^2} = 2\sqrt{5} = 2r$

$\therefore r = \sqrt{5}$ so $r^2 = 5$

midpoint = centre = $\left(\frac{-1+3}{2}, \frac{6+4}{2}\right) = (1,5)$

$\Rightarrow (x-1)^2+(y-5)^2 = 5$

Q4) $5^x = 17$

$\log(5^x) = \log(17)$

$x\log 5 = \log 17$

$x = \frac{\log 17}{\log 5} = \boxed{1.76}$

Q5a) $f(-2) = (-2)^3 + 4(-2)^2 - 2 - 6$
$= -8 + 16 - 8 = 0$

$\therefore (x+2)$ is a factor.

b)

$$\begin{array}{r l}
 & x^2 + 2x - 3 \\
x+2 & \overline{)\, x^3 + 4x^2 + x - 6} \\
 & x^3 + 2x^2 \\
 & \overline{0 \quad 2x^2 + x} \\
 & 2x^2 + 4x \\
 & \overline{0 \quad -3x - 6} \\
 & -3x - 6 \\
 & \overline{0 \quad 0}
\end{array}$$

$\therefore f(x) = (x+2)(x^2+2x-3)$

but $x^2+2x-3 = (x+3)(x-1)$

$\boxed{\therefore f(x) = (x+2)(x+3)(x-1)}$

5c) $(x+2)(x+3)(x-1) = 0$

$x+2=0$ $\boxed{x=-2}$

$x+3=0$ $\boxed{x=-3}$

$x-1=0$ $\boxed{x=1}$

Q6) $2(1-\sin^2 x) + 1 = 5\sin x$

$2 - 2\sin^2 x + 1 - 5\sin x = 0$

$-2\sin^2 x - 5\sin x + 3 = 0$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$

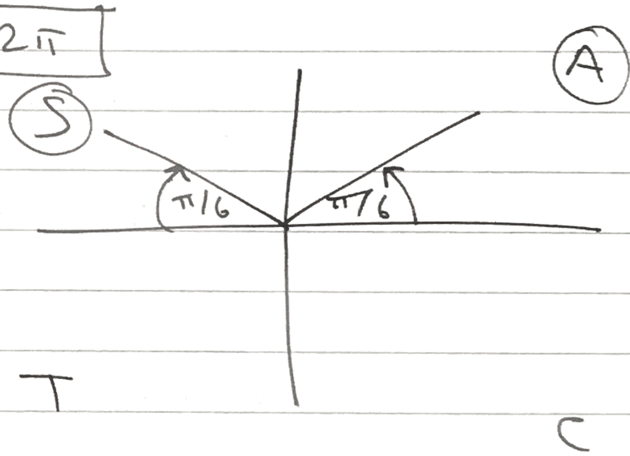
$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}$

$\sin x + 3 = 0$

$\sin x = -3$ × reject. no valid solutions ($\sin x \leq 1$)

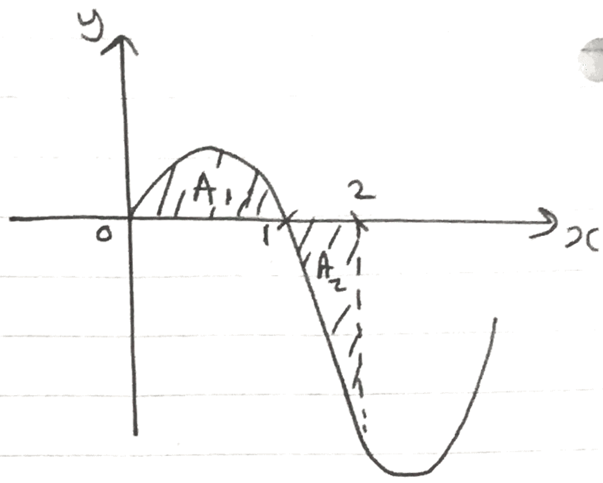
Solving in : $\boxed{0 \leq x < 2\pi}$



$x = \frac{\pi}{6}, \left(\pi - \frac{\pi}{6}\right)$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

Q7) Total Area = $A_1 + A_2$



$$A_1 = \int_0^1 [y]\,dx$$

$$A_2 = \int_1^2 [y]\,dx$$

$$A_1 = \int_0^1 [x(x-1)(x-5)]\,dx = \int_0^1 x(x^2-6x+5)\,dx$$

$$= \int_0^1 [x^3 - 6x^2 + 5x]\,dx = \left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1$$

$$= \left[\frac{1}{4} - 2 + \frac{5}{2}\right] = \frac{3}{4} \text{ units}^2$$

$$A_2 = \int_1^2 [x^3 - 6x^2 + 5x]\,dx = \left[\frac{x^4}{4} - 2x^3 + \frac{5}{2}x^2\right]_1^2$$

$$= [4 - 16 + 10] - \left[\frac{1}{4} - 2 + \frac{5}{2}\right] = \frac{-11}{4}$$

$$\therefore A_2 = \frac{11}{4} \text{ units}^2$$

$$A_1 + A_2 = \text{Total} = \frac{11}{4} + \frac{3}{4} = \boxed{\frac{7}{2}} \text{ units}^2$$

Q8a) $C = 1400v^{-1} + \frac{2}{7}v$

$$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7} = 0$$

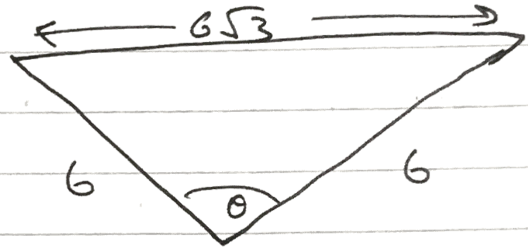
$$\frac{2}{7} = \frac{1400}{v^2}$$

$$v^2 = 4900 \quad \therefore v = \sqrt{4900} = \boxed{70 \text{ km/h}}$$

b) $\frac{d^2C}{dv^2} = 2800v^{-3} > 0$ (for all values of $v > 0$)

c) $C = \frac{1400}{(70)} + \frac{2}{7}(70) = \boxed{£40}$

Q9a) Cosine rule



$$\cos\theta = \frac{6^2+6^2-(6\sqrt{3})^2}{2(6)(6)}$$

$$\cos\theta = -\frac{1}{2} \quad \therefore \theta = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}\cdot 6^2 \cdot \frac{2\pi}{3} = \boxed{12\pi}$ m^2

c) Area $\Delta PQR = \frac{1}{2}ab\sin C = \frac{1}{2}(6)(6)\sin\left(\frac{2\pi}{3}\right) = \boxed{9\sqrt{3}}$ m^2

d) $\underbrace{12\pi}_{PQRS} - \underbrace{9\sqrt{3}}_{PQR} = PRS$ area $= \boxed{22.1 m^2}$

e) PR length $= r\theta = \frac{2\pi}{3} \times 6 = 4\pi$

$\therefore$ Perimeter $= 4\pi + 6 + 6 = \boxed{24.6 m^2}$

Q10a) $S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$ ~ ①

$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$ ~ ②

① − ② : $S_n - rS_n = a - ar^n$ [all terms cancel except a and ar^n]

$S_n(1-r) = a(1-r^n)$

$S_n = \dfrac{a(1-r^n)}{1-r}$ //

b) $\sum_{k=1}^{10} 100(2^k) \longrightarrow$ geometric series sum.

$a = 200$

$r = 2$

$200 + 200(2^1) + \dots + 200(2^9)$

$\therefore \sum_{k=1}^{10} 100(2^k) = S_{10} = \dfrac{200(1-(2)^{10})}{1-2} = \boxed{204600}$

c) $\dfrac{5}{6} \quad \dfrac{5}{18} \quad \dfrac{5}{54}$

$a \quad ar \quad ar^2$

$a = \dfrac{5}{6}$

$r = \dfrac{1}{3}$

$\left(\dfrac{5}{6} \times \dfrac{1}{3} = \dfrac{5}{18}\right)$

$S_\infty = \dfrac{a}{1-r} = \dfrac{\frac{5}{6}}{1-\frac{1}{3}} = \boxed{\dfrac{5}{4}}$

d) $|r| < 1$ (ie $-1 < r < 1$)